Longitudinal and transversal structure functions in two-dimensional electron magnetohydrodynamic flows

Kai Germaschewski and Rainer Grauer
Institut für Theoretische Physik I
Heinrich-Heine-Universität Düsseldorf
D-40225 Düsseldorf, Germany

Electron magnetohydrodynamic flows have recently attracted considerable interest not only in the field of collisionless reconnection but also as the first two-dimensional turbulent system showing Kolmogorov like scaling for the energy spectrum. Here, longitudinal and transversal structure functions are calculated for varying Reynolds numbers. The simulations show that the differences between longitudinal and transversal structure functions are finite size effects for this type of flow and vanish in the limit of high Reynolds numbers. In addition, the scaling of structure functions for velocity and current density could be described by assuming a log-Poisson statistics with two atoms, as recently proposed for passive scalar advection.

52.30.-q,52.35.Ra,52.65.Kj,02.30.Jr

I. INTRODUCTION

Some important magnetic reconnection phenomena occur on temporal and spatial scales which cannot be explained by models based on dissipation caused solely by collisions. Prominent examples are the earth's magnetosphere, where the aspect ratio of the current sheet is off by several orders of magnitude if one assumes a Sweet-Parker[1,2] model based on collisional resistivity. A second important example are sawtooth oscillations in fusion tokamak devices where the calculation of the collapse time is still a controversial issue[3–5].

To overcome this difficulty, in recent years there have been many studies that include electron inertia and the Hall term in Ohm's law[6–9]. In this paper, we concentrate on spatial scales below the ion inertial length c/ω_{pi} and time scales faster than the ion cyclotron frequency Ω_i. In this regime the ions form an immobile background and a fluid description for the electrons results in the electron magnetohydrodynamic (EMHD) equations[10]. Simulations show an enhanced reconnection rate even in two dimensions. Starting with smooth initial data thin current sheets form which are unstable to a Kelvin-Helmholtz instability. For sufficient low electron viscosity a turbulent state emerges as a superposition of very rare current sheets which will become Kelvin-Helmholtz unstable and the resulting monopolar vortices which are smoothly advected by the flow. A fascinating feature of this type of flow is the observation of Biskamp *et al.* [11] that the velocity fluctuations obey a Kolmogorov like energy spectrum despite the fact that the system is two-dimensional. In addition, it was shown by Celani *et al.*[12] that also a counterpart to the 4/5 law of three-dimensional Navier-Stokes turbulence holds.

Also still controversial is the issue whether high order longitudinal and transversal structure functions differ in the limit of infinite Reynolds number. In a study of three-dimensional Navier-Stokes turbulence Pelz and Boratav[13] observe differences between longitudinal and transversal structure functions. Similar observations were made by Grossmann *et al.*[14] In a first model, Chen *et al.*[15] related the differences of longitudinal and transversal structure functions to the probability distribution of vorticity and strain, respectively. Contrary to those findings are the observations by Camussi *et al.*[16] and Noullez *et al.*[17] that no significant differences between longitudinal and transversal structures functions occur. In a recent paper, Nelkin and Chen[18] claimed that no differences between longitudinal and transversal structure functions exist in the limit of infinite Reynolds number if a power law spectrum for pressure fluctuations holds. Two-dimensional electron magnetohydrodynamic turbulence is well suited to study this problem since it has many features in common with three-dimensional Navier-Stokes turbulence but with the computational advantage that much higher resolution can be achieved. We observe, that the difference between longitudinal and transversal structure functions are indeed a finite size effect. However, we point out that this does not need to be the generic case, since intermittency is weaker here than in three-dimensional Navier-Stokes turbulence.

Observing that the quantity $(1 - d_e^2\Delta)\psi$ is advected by the flow, where ψ is the magnetic flux function, we apply the approach of Dubrulle *et al.*[19–21] which was recently extended to the problem of passive scalar advection[22]. The numerically calculated structure functions for $(1 - d_e^2\Delta)\psi$ can be fitted to the prediction of this theory using one remaining free parameter. All other parameters of the theory are physically accessible.

II. NUMERICAL SIMULATIONS

The 2D EMHD equations[10,11] can be written in terms of two scalar potentials, the stream functions ϕ describing the poloidal electron flow $\mathbf{v} = \mathbf{e}_z \times \nabla\phi$ and the magnetic flux function ψ describing the poloidal magnetic field $\mathbf{B} = \mathbf{e}_z \times \nabla\psi$,

$$\partial_t \left(\phi - d_e^2\omega\right) - d_e^2\mathbf{v}\cdot\nabla\omega + \mathbf{B}\cdot\nabla j = -\mu(-\Delta)^\nu\phi, \quad (1)$$

$$\partial_t \left(\psi - d_e^2 j\right) + \mathbf{v}\cdot\nabla\left(\psi - d_e^2 j\right) = -\mu(-\Delta)^\nu\psi, \quad (2)$$

where $\omega = \Delta\phi$, $j = \Delta\phi$.

These equations were solved on a $2\pi \times 2\pi$ box with periodic boundary conditions. A pseudospectral method was used using up to 2048^2 collocation points. The dissipation terms were replaced by hyperviscous terms in order to extend the inertial range. Here, $\nu = 3$ was chosen. The equations were advanced in time by a standard leapfrog/Crank–Nicholson scheme. Dealiazing was performed using the 2/3 rule.

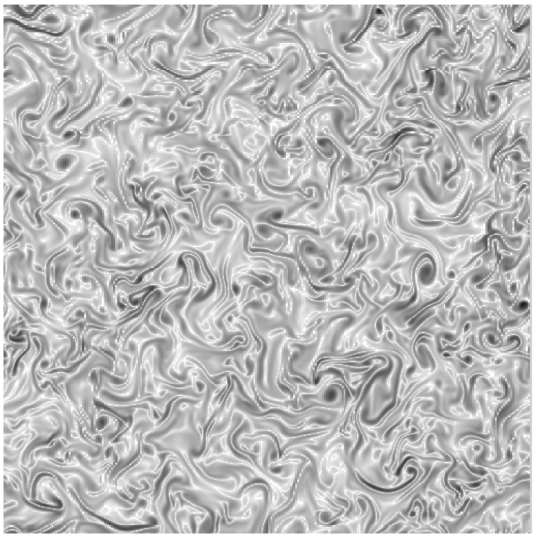

FIG. 1. $\phi - d_e^2\omega$ at time $t = 2.0$

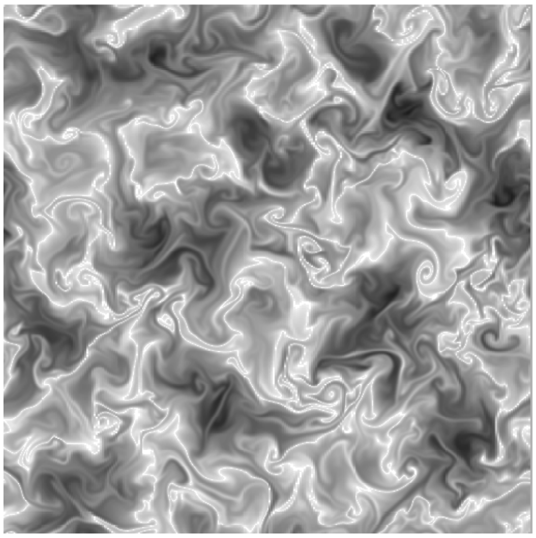

FIG. 2. $\psi - d_e^2 j$ at time $t = 2.0$

We studied freely decaying turbulence using the initial conditions proposed by Biskamp, Schwarz and Drake[11]: $\psi_\mathbf{k} \propto \exp\left(-k^2/2k_0^2 + i\alpha_\mathbf{k}\right), \phi_\mathbf{k} \propto \exp\left(-k^2/2k_0^2 + i\beta_\mathbf{k}\right)$, where $\alpha_\mathbf{k}, \beta_\mathbf{k}$ are random phases, i.e. the energy is concentrated on the large scales around k_0 which was set equal to 5 in our simulations.

After an initial relaxation to fully developed turbulence, the system remains in that quasi stationary state for a long time. The structure functions were determined from the numerical data in this state. A characteristic snapshot of the fields $\phi - d_e^2\omega$ and $\psi - d_e^2 j$ is shown in Figs. 1 and 2. As described in[11], turbulence is generated by the following mechanism: First, thin current sheets form and evolve until they are destroyed by a Kelvin-Helmholtz instability. Then, monopolar vortices result from the instability which are smoothly advected by the flow. As can be seen from Figs. 1 and 2 the current density is much smoother than the vorticity. The feedback of $\psi - d_e^2 j$ on the vorticity evolution in 1 is small but important. It destroys the conservation of enstrophy and therefore no inverse cascade of energy develops. Instead, a direct cascade of energy forms, which is why two-dimensional electron magnetohydrodynamic turbulence shares common features with three-dimensional Navier-Stokes turbulence on small scales.

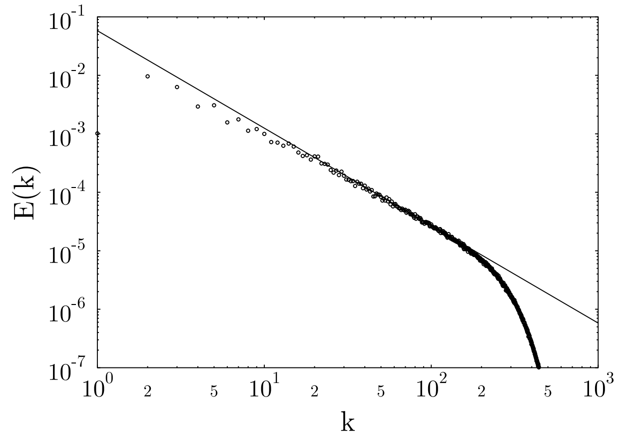

FIG. 3. Energy spectrum.

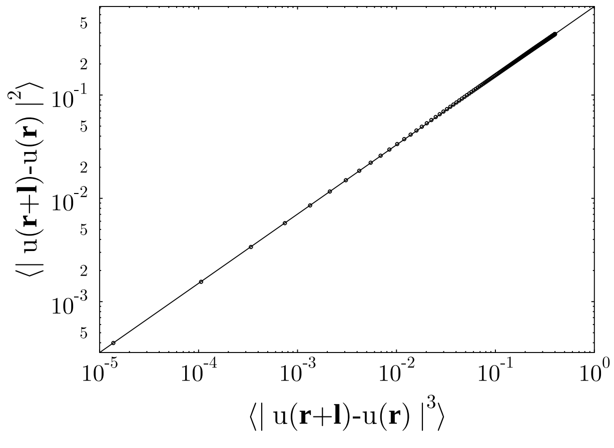

FIG. 4. ESS analysis: second order structure function, plotted through third order structure function.

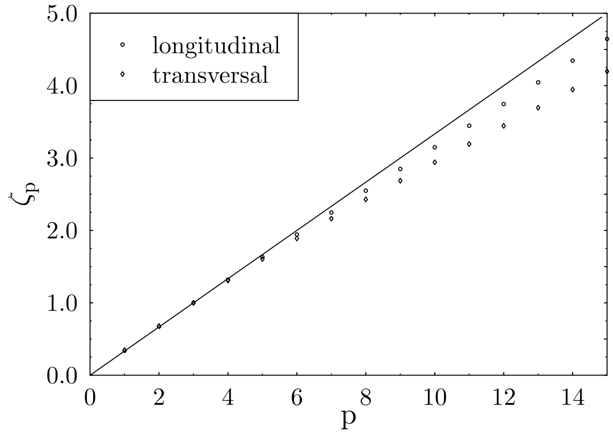

FIG. 5. Scaling exponents, resolution 512^2.

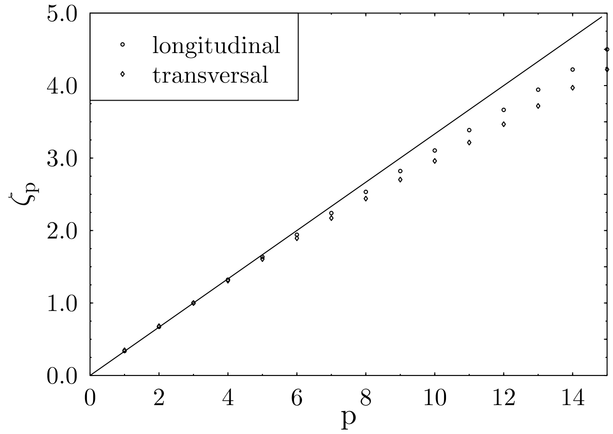

FIG. 6. Scaling exponents, resolution 1024^2.

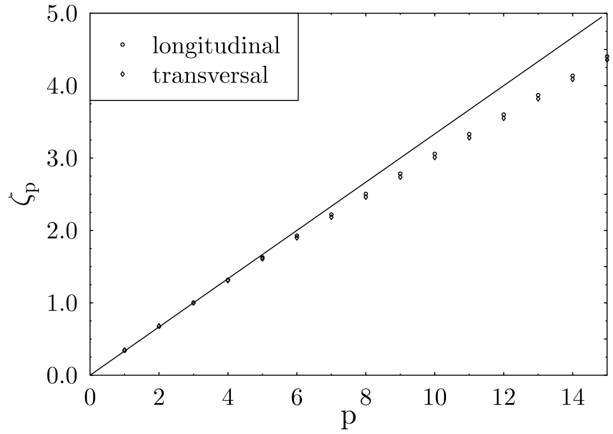

FIG. 7. Scaling exponents, resolution 2048^2.

To achieve higher resolution on the scaling exponents of the structure functions, the method of extended self similarity (ESS)[23] was applied.

Fig. 3 shows the energy spectrum. In the inertial range, the spectrum is well fitted by a power law with the exponent $-5/3$ (K41), indicated by the straight line. The low order structure functions ($p \leq 4$) show only little deviation from linear scaling, so that we use $\zeta_3 = 1$ as reference value for the use of ESS. This working assumption is also supported by the theory of Celani *et al.*[12], where the exact relation $\zeta_3 = 1$ is derived.

In Fig. 4, the second order structure function is plotted through the third order structure function in a logarithmic plot. It is obvious that ESS works very well for our results, proving the expected power law behavior for the structure functions.

The obtained scaling exponents are plotted in Fig. 5, 6 and 7 for different numerical resolutions of 512^2, 1024^2 and 2048^2 collocation points, respectively, correspondig to an increasing Reynolds number. Apparently, the deviating scaling exponents for longitudinal and transversal structure functions are an effect of finite Reynolds numbers, as they seem to collapse for a sufficiently high number of collocation points.

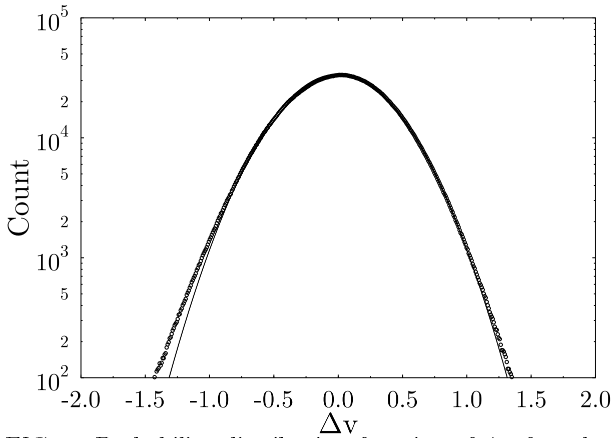

FIG. 8. Probability distribution function of Δv for a large displacement.

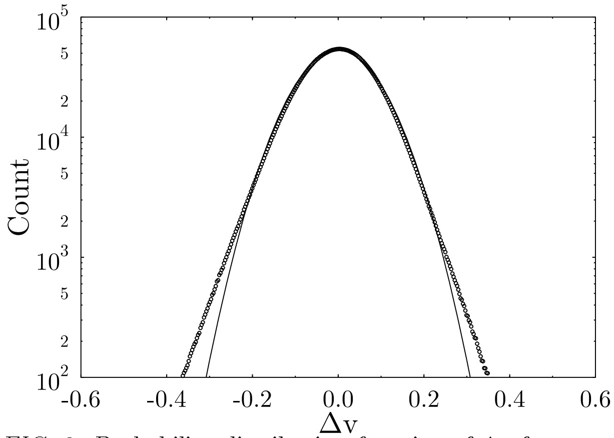

FIG. 9. Probability distribution function of Δv for a small displacement.

From a Kolmogorov–like theory (K41), one would expect a linear law for the scaling exponents $\zeta_p = p/3$. The numerical results show approximately this behavior for the low order structure functions. However, for increasing p there is a substantial deviation from this law, this so–called intermittency is expected and theoretically modelled later in this paper.

Intermittency can be explained by the fact that higher order structure functions tend to emphasize rare but strong gradients. Looking at the probability distribution functions (PDF) of velocity differences over large and small displacements, Figs. 8 and 9 respectively, this is confirmed.

For large differences, the PDF is basically Gaussian, in agreement with Kolmogorov's model. However, for smaller displacements, the PDF does not fall off as rapidly, showing a stretched exponential decay. Therefore, strong gradients are are not as rare as predicted by Kolmogorov's theory and lead to the deviation from the linear law for the scaling exponents.

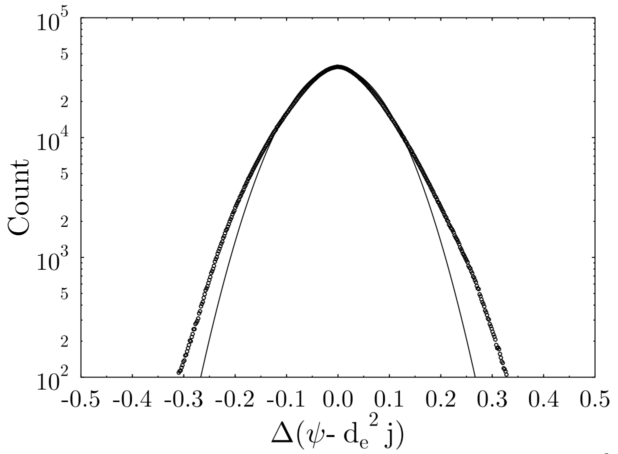

FIG. 10. Probability distribution function of $\Delta(\psi - d_e^2 j)$ for a large displacement.

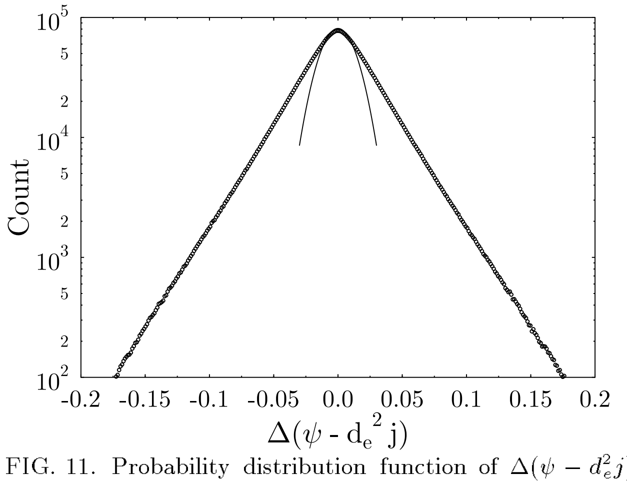

FIG. 11. Probability distribution function of $\Delta(\psi - d_e^2 j)$ for a small displacement.

This behavior is even more pronounced for the advected field $\psi - d_e^2 j$ (see Figs. 10 and 11). For a small displacement, the PDF shows a nearly exponential decay. Compared to the velocity structure functions, also stronger intermittency of the passive scalar is observed as will be presented later (see Fig. 13).

III. SCALE INVARIANT APPROACH

Rewriting the EMHD equations in terms of $F = \psi - d_e^2 j$ and $G = \phi - d_e^2 \omega$,

$$\partial_t F + \mathbf{v} \cdot \nabla F = \mu \nabla^2 \psi, \tag{3}$$

$$\partial_t G + \mathbf{v} \cdot \nabla G - \frac{1}{d_e^2} \mathbf{B} \cdot \nabla F = \mu \nabla^2 \phi, \tag{4}$$

the scalar F is advected by the velocity field $\mathbf{v}$ (and dissipated in the non-ideal case).

In the following model, we neglect the term $\frac{1}{d_e^2}\mathbf{B} \cdot \nabla F$. In the numerical simulations we found that $\frac{1}{d_e^2}\mathbf{B} \cdot \nabla F \ll \mathbf{v} \cdot \nabla G$. The main effect of this quantity is to destroy the additional conserved quantity and corresponding direct enstrophy cascade. Therefore, we will treat F as a passive scalar.

The scaling of the structure functions can be related to moments of scalar dissipation N_l and velocity dissipation ϵ_l using the *refined similarity hypothesis*[24–26]:

$$S_n^{\mathbf{v}}(l) = l^{n/3} \langle \epsilon_l^{n/3} \rangle,$$
$$S_n^F(l) = l^{n/3} \langle N_l^{n/2} \epsilon_l^{-n/6} \rangle. \tag{5}$$

So we need to know the scaling behavior of the moments $\langle \epsilon_l^{n/3} \rangle$ and $\langle N_l^{n/2} \epsilon_l^{-n/6} \rangle$ which will be evaluated in the following by means of two dimensional similarity transformations.

In this section we will apply the theory of Dubrulle and He[22] for passive scalar advection. In the following, we summarize their approach.

Dubrulle and Graner generalized the concept of units to measuring fields[20]. Like a physical quantity is described by a number multiplied by a unit, a random field ϕ is measured with respect to an appropriate reference field.

At each scale l and each point $\mathbf{x}$, an appropriate "unit" $R_l(\mathbf{x})$ is chosen. ϕ_l is obtained from the field ϕ by removing variations on scales smaller than l, e.g. by averaging over spheres with radius l. Then, the result of the measurement of ϕ_l with respect to the reference field R_l is given by $\{\phi_l(\mathbf{x})/R_l(\mathbf{x}) | \mathbf{x} \in \text{domain}\}$. Here we consider the case of homogeneous fields, so that the average $\langle \phi_l(\mathbf{x})/R_l(\mathbf{x}) \rangle$ is sufficient to describe the field ϕ at scale l.

Using the usual concept of units for physical quantities, it is always guaranteed, that the result of a measurement of a physical quantity does not depend on the choice of units. The approach of describing fields with respect to reference fields does not generally preserve this equivalence of units. But, in the case of scale invariant homogeneous fields, there always is a continuous set of reference fields, such that the equivalence principle is obeyed. This is because measurements made at one scale l are physically equivalent to measurements performed at another scale l' due to the assumption of scale invariance.

In the following, we deal with two scale invariant fields (ϕ, ψ). According to She and Leveque[27], the maximal events are defined by

$$\phi_l^\infty = \lim_{n\to\infty} \frac{\langle \phi_l^{n+1}\rangle}{\langle \phi_l^n\rangle} \quad , \quad \psi_l^\infty = \lim_{n\to\infty} \frac{\langle \psi_l^{n+1}\rangle}{\langle \psi_l^n\rangle} \tag{6}$$

and assumed to be finite. They obey a power law:

$$\phi_l^\infty \propto l^{\Delta\phi^\infty} \quad , \quad \psi_l^\infty \propto l^{\Delta\psi^\infty}. \tag{7}$$

A reference system (R_l^1, R_l^2, l_0) is used to measure (ϕ_l, ψ_l), the result is expressed in logarithmic variables X, Y, T:

$$X(T) = \ln\left\langle \frac{\phi_l}{R_l^1}\right\rangle \quad , \quad Y(T) = \ln\left\langle \frac{\psi_l}{R_l^2}\right\rangle \quad , \tag{8}$$

$$T = \ln\left(\frac{l}{l_0}\right) \quad . \tag{9}$$

Introducing the velocities with respect to the log–variables $U = dX/dT$, $V = dY/dT$, we obtain the scaling exponents of the fields:

$$\left\langle \frac{\phi_l}{R_l^1}\right\rangle \propto \left(\frac{l}{l_0}\right)^U \quad , \quad \left\langle \frac{\psi_l}{R_l^2}\right\rangle \propto \left(\frac{l}{l_0}\right)^V . \tag{10}$$

Suppose we measure the fields with respect to one reference system (R_l^1, R_l^2, l_0), here we obtain the quantities X, Y, T. If we use another system $(R_l'^1, R_l'^2, l_0')$, we naturally obtain different results X', Y', T'. Suppose the two systems are equivalent (which we know to hold for scale invariant fields), there needs to exist a transformation $M_{R/R'}$, which maps (X, Y, T) to (X', Y', T'), corresponding to a change of units.

These transformations M form a group, where the needed operation is the composition of two transformations.

Under the additional assumption, that this group is commutative, which is suggested from the normal changing of units, the structure of the transformation group is completely determined. There are two parameters, $U_{R/R'}$, $V_{R/R'}$, which may be thought of as the velocity of R with respect to R'.

In this picture, the procedure is closely related to the theory of special relativity, where the transformation group is not commutative, though. In particular, composing two transformations from reference fields R to R' and from R' to R'', the parameters are transformed according to

$$\begin{aligned}&(U_{R/R''}, V_{R/R''}) = \\ &(U_{R/R'}, V_{R/R'}) \otimes (U_{R'/R''}, V_{R'/R''})\end{aligned} \tag{11}$$

with the composition law $\otimes$:

$$\begin{aligned}U'' &= \frac{U + U' - aUU' - dVV' - j(U'V + UV')}{1 - hUU' - kVV'}, \\ V'' &= \frac{V + V' - cjUU' - bVV' - cd(U'V + UV')}{1 - hUU' - kVV'}.\end{aligned} \tag{12}$$

a, b, c, d, h, j, k are parameters that depend on the physical model. The exact derivation is given in[22].

In our effort to evaluate moments in the *refined similarity hypothesis*, we choose the class of reference systems to be

$$R_l = (N_l\epsilon_l, \epsilon_l), \tag{13}$$

$$R_l'(p', q') = \left(N_l^{1-p'}(N_l^\infty)^{p'}\epsilon_l^{1-q'}(\epsilon_l^\infty)^{q'}, \epsilon_l^{1-q'}(\epsilon_l^\infty)^{q'}\right), \tag{14}$$

$$R_l''(p'', q'') = \left(N_l^{1-p''}(N_l^\infty)^{p''}\epsilon_l^{1-q''}(\epsilon_l^\infty)^{q''}, \epsilon_l^{1-q''}(\epsilon_l^\infty)^{q''}\right), \tag{15}$$

Measuring $\langle R_l/R_l'\rangle$, $\langle R_l/R_l''\rangle$ and $\langle R_l'/R_l''\rangle$, we obtain the exponents $U_{R/R'}, V_{R/R'}$, $U_{R/R''}, V_{R/R''}$ and $U_{R'/R''}, V_{R'/R''}$.

Now we need to know the exact form of the composition law (12). Since the scalar ψ is passive, V needs to be independent of U. Furthermore, we assume log–Poisson statistics for the velocity increments[20], equivalent to She and Leveque's assumptions. This leads to the simplified composition law

$$\begin{aligned}U(p'', q'') &= U(p''-p', q''-q') + U(p', q') \\ &\quad -aU(p''-p', q''-q')U(p', q') \\ &\quad -dV(q''-q')V(q') \\ &\quad -j(U(p''-p', q''-q')V(q') \\ &\quad +U(p', q')V(q''-q')), \\ V(q'') &= V(q''-q') + V(q') - bV(q''-q')V(q').\end{aligned} \tag{16}$$

For $p' = p'' = q' = q'' = 0$, there are two solutions:

$$U_+ = \frac{b-j}{ab} \quad , \quad V_* = \frac{1}{b}, \tag{17}$$

$$U_- = -\frac{j}{ab} \quad , \quad V_* = \frac{1}{b}. \tag{18}$$

Using these denominations, the general solution of (16) can be expressed as

$$\begin{aligned}U(p, q) &= (U_+ - U_-)(1 - \beta_1^p) + V_*(1 - \beta_2^q) \\ &\quad +(U_- - V_*)(1 - \beta_1^p)(1 - \beta_2^q),\end{aligned} \tag{19}$$

$$V(q) = V_*(1 - \beta_2^q). \tag{20}$$

Choosing the reference fields as stated above, we obtain the following result:

$$\left\langle \frac{R_l^1}{R_l'^1}\right\rangle = \frac{1}{(N_l^\infty)^{p'}(\epsilon_l^\infty)^{q'}}\left\langle N_l^{p'}\epsilon_l^{q'}\right\rangle \propto l^{U_{R/R'}}, \tag{21}$$

$$\left\langle \frac{R_l^2}{R_l'^2}\right\rangle = \frac{1}{(\epsilon_l^\infty)^{q'}}\left\langle \epsilon_l^{q'}\right\rangle \propto l^{V_{R/R'}}. \tag{22}$$

Therefore, we found the scaling of the moments to be

$$\langle N_l^p \epsilon_l^q \rangle \propto l^{p\cdot\Delta N^\infty + q\cdot\Delta\epsilon^\infty + U(p,q)}, \tag{23}$$

$$\langle \epsilon_l^q \rangle \propto l^{q\cdot\Delta\epsilon^\infty + V(q)}. \tag{24}$$

Rewriting this, the scaling exponents for the velocity field ζ and for the passive scalar ξ have been determined to be

$$\xi_n = \left(\frac{1}{3} + \frac{\Delta N^\infty}{2} - \frac{\Delta\epsilon^\infty}{6}\right) n + U\left(\frac{n}{2}, -\frac{n}{6}\right), \tag{25}$$

$$\zeta_n = \left(\frac{1}{3} + \frac{\Delta\epsilon^\infty}{3}\right) n + V\left(\frac{n}{3}\right). \tag{26}$$

The parameters U_+, U_-, V_* can be found considering the limits $p \to \infty$ and $q \to \infty$, respectively. Let τ_q be the scaling exponents of the moments of velocity dissipation, i. e. $\langle \epsilon_l^q \rangle \propto l^{\tau_q}$, then for $q \to \infty$ follows: $\tau_q = q\cdot\Delta\epsilon^\infty + V_*$. So it makes sense to choose V_* to be the codimension of the most dissipative structures. In a similar way, $U_+ - U_-$ can be determined to be the codimension of the most dissipative structures of the scalar. In EMHD, these structures are layers. Neglecting the coupling term $U_- - V_*$ as in[22], we end up with

$$V_* = 1 \ , \quad U_+ - U_- = 1 \ , \quad U_- - V_* = 0. \tag{27}$$

From the exact relation $\zeta(3) = 1$ which is equivalent to $\langle \epsilon_l \rangle \propto l^0$ we obtain

$$\beta_2 = 1 + \Delta\epsilon^\infty. \tag{28}$$

The same way we find

$$\beta_1 = 1 + \Delta N^\infty. \tag{29}$$

In the end, we are left with two undetermined parameters $\Delta\epsilon^\infty$ and ΔN^∞ in this model, which were used to fit this theory to the numerical results.

As can be seen in the Figures 12 and 13, using only one free parameter per plot, the numerical data are fitted very well by the presented theory.

However, it would of course be highly desirable to obtain values for $\Delta\epsilon^\infty$ and ΔN^∞ from theoretical considerations instead of numerics.

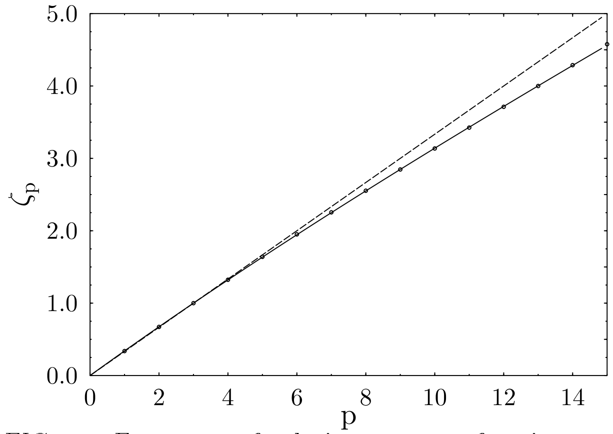

FIG. 12. Exponents of velocity structure functions: numerical results (points) and prediction of theory (solid), K41 (dashed).

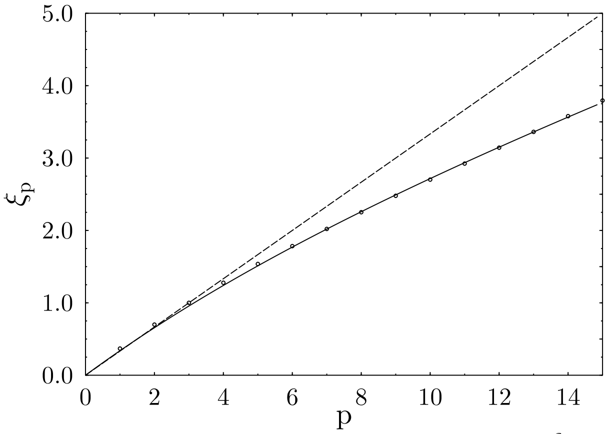

FIG. 13. Exponents of structure functions for $\psi - d_e^2 j$: numerical results (points) and prediction of theory (solid), K41 (dashed).

IV. CONCLUSIONS

In this paper we studied the intermittency properties of electron magnetohydrodynamic flows. It was shown that in this type of flow the difference between longitudinal and transversal structure functions is a finite size effect. We emphasize that this need not to be the general case since the dissipative structures are much less anisotropic than e.g. vortex tubes in three dimensional Navier-Stokes turbulence. However, we want to point out that the crossover to a common scaling of longitudinal and transversal structure functions appeared at a resolution much higher that 512 collocation points in each spatial direction. One definitely has to keep this in mind in interpreting current three dimensional numerical simulations.

In addition, the treatment of passive scalar advection introduced by Dubrulle and He[22] could describe the intermittency properties of velocity and current density fluctuations with only one fitting parameter for each field. A major challenge in improving this approach will be to relate these parameters to intrinsic features of the underlying flow.

ACKNOWLEDGMENTS

We like to acknowledge discussions with Bérengère Dubrulle who made the preprint[22] available to us. We thank K. H. Spatschek for his continuous support. This work was performed under the auspices of the Sonderforschungsbereich 191.

[1] P. A. Sweet, Nuov Cimento Suppl. **8**, 188 (1958).
[2] E. N. Parker, Astrophys. J. Suppl. **8**, 177 (1963).
[3] D. Biskamp, *Nonlinear Magnetohydrodynamics*, *Cambridge Monographs on Plasma Physics* (Cambridge University Press, Cambridge, 1993).
[4] D. Biskamp, Phys. Plasmas **4**, 1964 (1997).
[5] J. Aparicio, M. G. Haines, R. J. Hastie, and J. P. Wainwright, Phys. Plasmas **5**, 3180 (1998).
[6] S. V. Bulanov, F. Pegoraro, and A. S. Sakharov, Phys. Fluids B **4**, 2499 (1992).
[7] M. Ottaviani and F. Porcelli, Phys. Rev. Lett. **71**, 3802 (1993).
[8] D. Biskamp, E. Schwarz, and J. F. Drake, Phys. Rev. Lett. **75**, 3850 (1995).
[9] D. Biskamp, E. Schwarz, and J. F. Drake, Phys. Plasmas **4**, 1002 (1997).
[10] A. S. Kingsep, K. V. Chukbar, and V. V. Yan'kov, in *Electron Magnetohydrodynamics*, edited by B. Kadomtsev (Consultants Bureau, New York, 1990), Vol. 16, pp. 243–291.
[11] D. Biskamp, E. Schwarz, and J. F. Drake, Phys. Rev. Lett. **76**, 1264 (1996).
[12] A. Celani, R. Prandi, and G. Boffetta, Europhys. Lett. **41**, 13 (1997).
[13] O. N. Boratav and R. B. Pelz, Phys. Fluids **9**, 1400 (1997).
[14] S. Grossmann, D. Lohse, and A. Reeh, Physics of Fluids **9**, 3817 (1997).
[15] S. Chen, K. R. Sreenivasan, M. Nelkin, and N. Cao, Phys. Rev. Lett. **79**, 2253 (1997).
[16] R. Camussi, D. Barbagallo, G. Guj, and F. Stella, Phys. Fluids **8**, 1181 (1996).
[17] A. Noullez *et al.*, J. Fluid Mech. **339**, 287 (1997).
[18] M. Nelkin and S. Chen, Phys. Fluids **10**, 2119 (1998).
[19] B. Dubrulle, Phys. Rev. Lett. **73**, 959 (1994).
[20] B. Dubrulle and F. Graner, J. Phys. II France **6**, 797 (1996).
[21] B. Dubrulle and F. Graner, J. Phys. II France **6**, 817 (1996).
[22] B. Dubrulle and G. W. He, "Possible Statistics of Two Coupled Random Fields. Application to Passive Scalar," submitted to Phys. Rev. E.
[23] R. Benzi *et al.*, Europhys. Lett. **24**, 275 (1993).
[24] A. N. Kolmogorov, J. Fluid Mech. **13**, 82 (1962).
[25] A. M. Obukhov, Izv. Akad. Nauk. SSSR, Ser. Geogr. Geofiz. **13**, 58 (1949).
[26] S. Corrsin, J. Appl. Phys. **22**, 469 (1951).
[27] Z.-S. She and E. Leveque, Phys. Rev. Lett. **72**, 336 (1994).