ON THE DYNAMICS OF THE SOLAR CORONA: FIRST RESULTS OBTAINED WITH A NEW 3D MHD MODEL

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ABSTRACT

A newly developed self-consistent 3D MHD code (see poster by Kleimann et al., this conference) is applied to the problem of the dynamics of the solar corona. First, we present the basic system of equations for a two-fluid description of the solar wind plasma and point out possible numerical difficulties arising from an improper choice of variables. Second, we perform a study of the solar wind expansion during phases of minimum solar activity, serving mainly as a first 'real world' test case. Third, we discuss first results of the application of the model to propagating disturbances, such as coronal mass ejections and/or shocks.

Key words: solar wind; MHD; numerical simulation; two-fluid modeling; CWENO.

1. MOTIVATION

For many decades space scientists have been attracted to the solar wind and its delicate interaction with the highly variably solar magnetic field. A recent wide-spread increase in interest was spawned not only by many stimulating inputs due to ground-breaking space-based observations but also by the fact that the impact of space weather phenomenae on the Earth's environment is becoming more obvious every year. Thus, detailed and realistic models of solar wind dynamics have in fact become mandatory for space weather forecasting and data analysis of forthcoming spacecraft missions to the inner heliosphere.

Fortunately, these urgently needed advances have also become possible in the first place by recent progress in computing speed and sophisticated algorithms.

Clearly, such a huge and complex topic has room for many investigators, and a number of groups have indeed achieved impressive results in this field. It therefore seems appropriate to begin by pointing out why we felt that the community might benefit from yet another modeling approach.

2. WHY ANOTHER MODEL ?

The new aspects by which our approach differs from various other works can be grouped into two fields: physics and numerics.

2.1. New aspects – physics

The novel aspects on the physics side are mainly three-fold:

- Contrary to previous assumptions, the SOHO mission has revealed that the electrons and protons which largely constitute the solar wind plasma are indeed far away from thermal equilibrium, with the proton temperature being about five times larger than the electron temperature. For this reason, it seems appropriate to abandon the single-fluid model approach and instead consider these species as two separate fluids interacting via collisions.
- The process of restructuring of magnetic field lines known as *magnetic reconnection* has been identified to play a major role in both coronal heating and the dynamic evolution of "disturbances", such as coronal mass ejections. Since this process cannot occur in an ideally conducting medium, a finite resistivity is explicitly included into our system of equations. As a side effect, this will also help to improve the code's numerical stability.

- Finally, we will try to abandon the concept of both ad-hoc heating functions (which do help to generate reasonable wind speeds but completely lack any physical mechanism) and consistent heating rates according to Hollweg (1986) in favor of true Alfvénic wave heating. The fluctuations of the interplanetary magnetic field (mainly Alfvén waves) make up a considerable fraction of the magnetic field energy in the inner heliosphere, and they could well be partly responsible for heating the corona to its temperature of some 10^6 K.

2.2. New aspects – numerics

Since our object of study, the solar corona at full maximum, is clearly void of any potentially simplifying symmetry, one major point in favor of spherical or cylindrical coordinates is lost. We therefore decided to use a plain Cartesian grid, which does make the treatment of the photospheric boundary slightly more complicated but is much simpler to implement, and also advantageous in terms of numerical stability.
To advance the equations in this box-shaped computational domain, we have implemented a newly developed CWENO (*Central Weighted Essentially Non-Oscillatory*) algorithm. (Nessyahu & Tadmor, 1990 and Kurganov & Levy, 2000) Since its details are given in our poster contribution *On the dynamics of the solar corona: the numerics behind a self-consistent 3D MHD model* by Kleimann et al. (this conference), we only note here that CWENO is a

- third-order, (almost) non-oscillatory
- flux-conserving
- shock-capturing

scheme, and that all these characteristics make it of course a very appealing tool for our purpose.
This algorithm is then combined with two powerful numerical features which have recently received increased attention within the computational fluid dynamics community:

1. *Adaptive mesh refinement (AMR).* High spatial grid resolution, while being not only desirable but indeed necessary for our envisaged simulations, is crucially limited by the available resources in terms of both computational memory and CPU speed. We have therefore chosen to implement a powerful adaptive mesh refinement technique, which dynamically adapts the local spatial resolution according to the presence or absence of strong gradients. Details of this procedure are described by Friedel, Grauer, and Marliani (1997).
2. An implementation of a recent *divergence cleaning* algorithm by Dedner et al. (2002) ensures that the solenoidality condition $\nabla \cdot \mathbf{B} = 0$ stays valid throughout the simulation, thus preventing the rise of increasingly unphysical field line configurations which would otherwise be created by the accumulation of round-off errors.

3. BASIC EQUATIONS

In dimensionless form, the equations governing the evolution of our magnetized two-fluid system (characterized by densities ρ_α, velocities $\mathbf{v}_\alpha$ and gas pressures p_α, resistivity η, magnetic induction $\mathbf{B}$ and associated current density $\mathbf{j} = \nabla \times \mathbf{B}$) is given by

$$\begin{aligned}
\partial_t\ \rho &= -\nabla \cdot (\rho \mathbf{v}) && (1)\\
\partial_t(\rho\ \mathbf{v}) &= -\nabla \cdot (\rho\mathbf{v}\ \mathbf{v}) - \nabla(p_\mathrm{e} + p_\mathrm{i} + p_\mathrm{w}) && (2)\\
&\quad -\rho\ \Phi_\mathrm{grav} + \mathbf{j} \times \mathbf{B} &&\\
\partial_t\ \mathbf{B} &= -\nabla \cdot (\mathbf{v}\ \mathbf{B} - \mathbf{B}\ \mathbf{v}) - \nabla\Psi && (3)\\
&\quad +\eta\ \nabla^2\mathbf{B} - \nabla\eta \times \mathbf{j} &&\\
\partial_t\ p_\alpha &= -\nabla \cdot (p_\alpha\ \mathbf{v}_\alpha) - \nu_\mathrm{ep}(p_\alpha - p_\beta) && (4)\\
&\quad -(\gamma - 1)\left[p_\alpha(\nabla \cdot \mathbf{v}_\alpha) + Q_\alpha\right] &&\\
\partial_t\ p_\mathrm{w} &= -\nabla \cdot \left[p_\mathrm{w}(\mathbf{v} + \mathbf{v}_\mathrm{A})\right] - Q_\mathrm{p} && (5)\\
&\quad +(\nabla \cdot \mathbf{v})\ (p_\mathrm{w}/2) &&\\
\partial_t\Psi &= -\nabla \cdot \left(c_h^2\ \mathbf{B}\right) - (c_h/c_p)^2\ \Psi && (6)
\end{aligned}$$

where α and $\beta \neq \alpha$ indicate the different species (e for electrons, p for protons). The equation of state $p_\alpha = \rho_\alpha\ T_\alpha$ of a mono-atomic ideal gas is adopted, along with an adiabatic exponent $\gamma = 5/3$. We also introduced the mean density $\rho := \rho_\mathrm{p} + \rho_\mathrm{e}$ (assuming quasi-neutrality, i.e. $\rho_\mathrm{e}/m_\mathrm{e} = \rho_\mathrm{p}/m_\mathrm{p}$) and the mean velocity

$$\mathbf{v} := \frac{(\rho_\mathrm{p}\mathbf{v}_\mathrm{p} + \rho_\mathrm{e}\mathbf{v}_\mathrm{e})}{\rho}\ , \tag{7}$$

from which the expressions for the individual velocities may be obtained by

$$\mathbf{v}_\mathrm{e} = \mathbf{v} + \frac{\mathbf{j}}{\rho} \quad \text{and} \quad \mathbf{v}_\mathrm{p} = \mathbf{v} - \left(\frac{m_\mathrm{e}}{m_\mathrm{p}}\right)\frac{\mathbf{j}}{\rho} \approx \mathbf{v}\ . \tag{8}$$

As can be seen on the rhs. of the momentum balance equation, plasma elements are accelerated by gradients of both gas and wave pressures, as well as magnetic induction and the Sun's gravitational potential $\Phi_\mathrm{grav} = -GM_\mathrm{Sun}/r$. The induction equation (3) not only accounts for spatial variation of η via the $\nabla\eta \times \mathbf{j}$ term but is furthermore supplemented with a new term $-\nabla\Psi$ accounting for said divergence cleaning, Ψ itself being an non-physical quantity which carries away any non-vanishing $\nabla \cdot \mathbf{B}$. For further details of Ψ and its equation (6) see Dedner et al. (2002).
The gas pressures are advanced in time using Eqs. (4), with ν_ep denoting the frequency at which elastic collisions occur.
Electrons are heated by standard Ohmic dissipation

$$Q_\mathrm{e} = \eta\ \mathbf{j}^2\ , \tag{9}$$

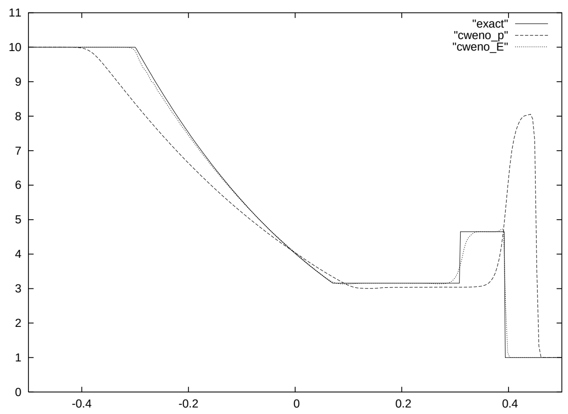

Figure 1. The Sod problem. Two adjacent domains of different gas pressures and densities are initially separated by a membrane. As the membrane is removed, the gas expands freely, thus giving rise to various types of traveling discontinuities. Using the total energy (which in this case is a conserved quantity), the computed density profile (dotted line) closely resembles the exact solution (solid line). A comparison run using gas pressure as variable yields unacceptable deviations (dashed line), which do not vanish as the spatial resolution is refined.

while the protons experience Alfvénic wave heating via

$$Q_{\rm p} = -\left[f_{\rm H}\ln(f_{\rm H}/f_0)\right]^{-1} p_{\rm w}\,(\mathbf{v}+\mathbf{v}_{\rm A})\cdot\nabla f_{\rm H}\ , \quad (10)$$

where $\mathbf{v}_{\rm A} := \mathbf{B}/\sqrt{\rho}$ denotes the Alfvén speed, the idea being that magnetic waves of frequency f_0 are generated at the photosphere by convective plasma motions, then travel spacewards while being shifted to higher frequencies by non-linear wave-wave interaction until they reach an upper frequency $f_{\rm H}$ (taken to be a fixed fraction of the proton cyclotron frequency) and get dissipated. The evolution equation (5) for the wave pressure $p_{\rm w}$ is based on a work by Tu, Pu, & Wei (1984), in which the time-independent version of (5) is derived. The functional shape of the wave heating term (10) then follows from the constraint that the system's total energy be conserved.

There is, however, one problem associated with this approach: Since differential equations like (1) are derived from integral conservation laws under the assumption of smooth solutions, one cannot guarantee that in the presence of shocks (i.e. discontinuous solutions) *any* numerical scheme will converge to the "right" solution unless it is formulated in terms of a conserved quantity.[1] This behavior is illustrated in Fig. 1 which shows the density profile computed for Sod's classical Riemann problem (Sod 1978) using different physical quantities as variables.

But as both species of our solar wind model can (and will) exchange energy via collisions, their respective total energies are no longer conserved. We therefore lack one more conserved quantity besides the system's total energy.[2] From our current point of view, it thus seems that one cannot properly describe shocks in a two-fluid model, which if true would be a pity.

4. FIRST RESULTS

As with every new numerical code, an extensive test phase has to precede any scientific application. Selected test results are reported in the above mentioned poster contribution and will thus not be reproduced here.

Two particular solar wind-related simulation runs appear worth noting:

1. A hydrodynamic, initially isothermal Parker wind solution (Parker 1958) is subjected to axially symmetric heating

$$Q(r,\theta) \;\propto\; \frac{1}{r^2}\exp\left[-1.4\left(\frac{r}{R_{\rm Sun}}-1\right)\right]\times \times\,(1+w\cos\theta) \quad (11)$$

   (r and θ being spherical, Sun-centered coordinates), and then allowed to relax into a stationary state. The dependence of Q on the angular variation term can be adjusted by varying the parameter w. The final configuration for the case $w = 0.5$ is shown in Fig. 2. More pronounced variations in the resulting temperature distribution have been obtained using correspondingly higher values of w (not shown).

2. In order to see how this configuration responds to a time-dependent disturbance, density and velocity were modified in order to model a very coarse approximation to a CME. Fig. 3 shows a time sequence of corresponding density contours. It can clearly be seen how the perturbation propagates outwards while its density decreases.

5. SUMMARY AND OUTLOOK

We have reported on first steps of a novel approach to model the evolution of solar wind dynamics in three dimensions using a combination of recently developed numerical methods and an improved set of equations for a two-fluid model for the heliospheric plasma. The importance of a proper choice of integration variables is pointed out by means of the well-known Sod problem. Moreover, we could show first hydrodynamic results of the application of our new code to the solar wind. Next steps will be modeling using more realistic parameters and, of course, the inclusion of magnetic field effects.

[1] For a comprehensive treatment of this phenomenon see, e.g., chapter 12 of (LeVeque, 1992).

[2] The entropy density $s = p/\rho^{\gamma-1}$ cannot be used here since its value is not conserved across shock surfaces either.

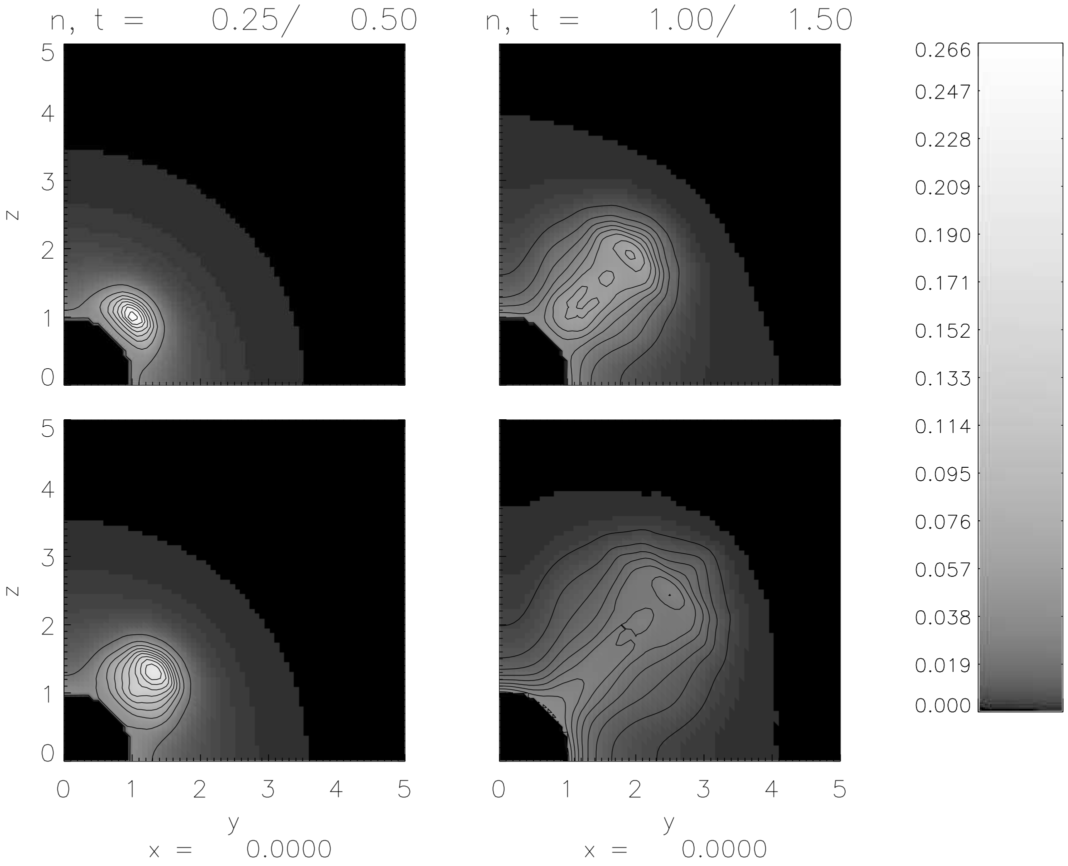

Figure 3. Time sequence of a propagating disturbance (shading is density).

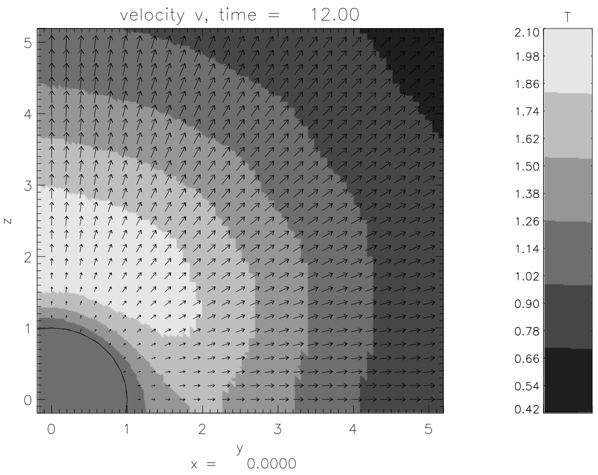

Figure 2. Steady state distribution of temperature (shading) and velocity (arrows) in arbitrary units.

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REFERENCES

Dedner, A. et al., 2002, J. Comp. Phys. 175, 645

Friedel, H., Grauer, R., Marliani, C., 1997, J. Comp. Phys. 134, 190

Hollweg, J., 1986, J. Geophys. Res. 91, 4111

Kurganov, A., Levy, D., 2000, SIAM J. Sci. Comp. 22, 1461

LeVeque, R., 1992, Numerical Methods for Conservation Laws, Birkhäuser

Nessyahu, H., Tadmor, E., 1990, J. Comp. Phys. 87(2), 408

Parker, E., 1958, Astrophys. J. 128, 664

Sod, G., 1978, J. Comp. Phys. 27, 1